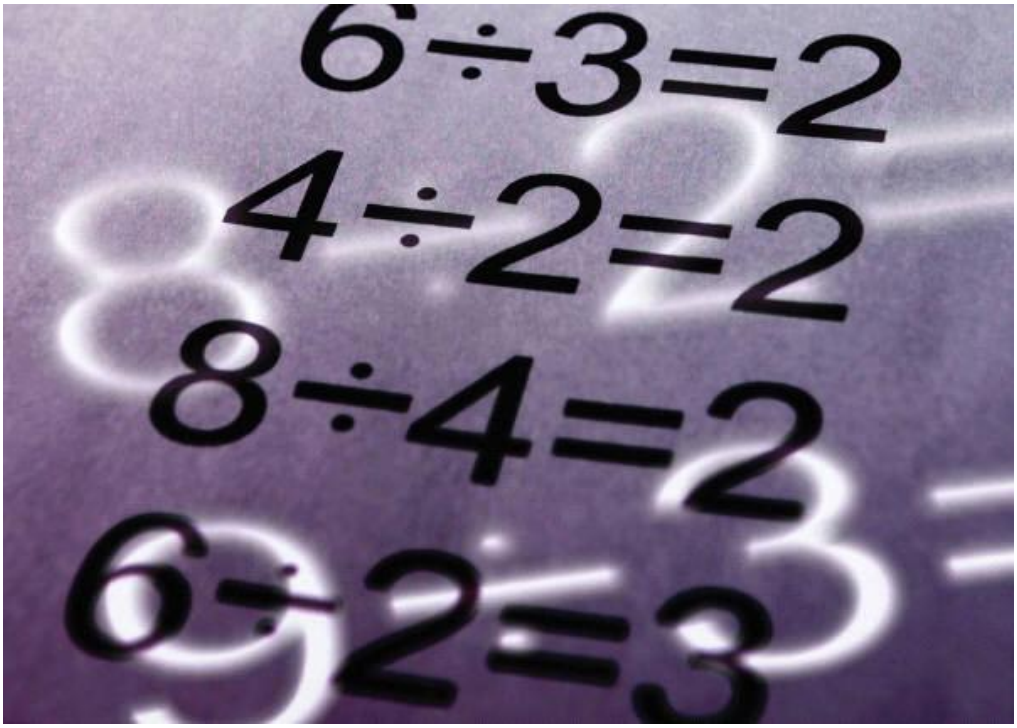
A support document for the teaching of division for maths co-ordintors.

PARTNERSHIP IN LEADERSHIP & LEARNING



Enhancing Understanding of Division in LKS2

In order to facilitate children's understanding of division effectively, it is essential that teachers are aware of the two different conceptual structures of division; the range of language and contexts associated with division; and how and when resources and representations, mental and written methods can best be used.

Sharing and Grouping

In lower KS2 we usually use two structures when talking about division. These are 'sharing equally between' and 'grouping into equal sets.' To understand this fully it is easiest to look at how these are represented:

$$12 \div 3 = 4$$

Sharing

Lucy has 12 sweets and shares them between 3 of her friends. How many sweets does each friend get?

We would say:

12 sweets shared between 3 people gives 4 sweets each.

The answer lies in the value of each equal share.

Grouping

Lucy has 12 sweets. She wants to put them into bags of 3. How many bags will she have?

We would say:

12 sweets put into bags (groups) of 3 gives 4 bags (groups).

The answer lies in the number of equal groups.

The grouping structure of division also shows clearly that division is the inverse of multiplication. For example, since 4 x 3 = 12, then 12 ÷ 3 must be 4. The division by 3 'undoes' the effect of multiplying by 3 (Haylock, 2006). So to solve the bags of sweets problem, we would need to ask,

'How many 3s are there in 12?' or 'What do I multiply 3 by to get 12?'

As teachers we need to understand these structures and make them explicit to the children by talking about them, and providing opportunities to explore both in a concrete manner, linking the physical model to the symbolic number sentence.

We also need to be very aware of the language we are using so that we don't talk about sharing when we are grouping or vice versa.

In upper KS2 you should also begin to use the ratio/scaling model for division. (See Haylock, 2006 for a full explanation).

Division and Multiplication

Arrays are invaluable in enhancing children's understanding of division because they help children to connect the two operations using a strong visual image. We recommend allowing children to explore the connection between multiplication and division, using arrays both enactive (concrete materials) and iconic (drawings and pictures) representations, as the starting point for any learning about division.

Arrays make no distinction between grouping and sharing, and offer 6 interpretations involving multiplication and division.

3 groups of 5	5 x 3 = 15
5 groups of 3	3 x 5 = 15
15 split into 3 equal shares	15 ÷ 3 = 5 (5 in each set)
15 split into 5 equal shares	15 ÷ 5 = 3 (3 in each set)
15 split into groups of 3	15 ÷ 3 = 5 (5 equal groups)
15 split into groups of 5	15 ÷ 5 = 3 (3 equal groups)

The Language and Contexts of Division

Children often reach Y6 and still find it difficult to identify division within word problems. To address this we need to ensure that children are exposed to the wide range of language and real life contexts associated with division. This does not mean simply responding to cue words such as 'share' and 'groups of.'

There are at least 6 categories of division word problems (Bird, 2009):

1. Problems about equivalent groups;
   Alex bought 30 pencils in 5 boxes. How many pencils are in each box?

2. Problems about equivalent measures;
   If four identical parcels are tied up with ribbon, how much ribbon could you use on each parcel out of a 2m strip?

3. Rate problems;
   A car travels at 20 miles per hour. How long would it take to cover 100 miles?

4. Problems about area or rectangular arrays;
   How many rows of 12 chairs with 180 chairs?

5. Scalar problems;
   Dad is 48 and is exactly 6 times as old as Sam. How old is Sam?

6. Problems involving Cartesian products;
   By combining sizes and colours, a factory produces 12 different versions of stripy t-shirt. The t-shirts come in small, medium or large. How many colours are there in each size?

- Which of these questions are sharing equally between and which are grouping?
- Which contexts could lend themselves to either structure?
- How could they be modeled as an array?

Remainders

The idea of a remainder is best taught by first using concrete materials and visual images, and with small numbers. In the context of real life and word problems, it is imperative that children understand when to round up and when to round down. To facilitate this understanding, children must be exposed to a wide range of real life problems and scenarios which prompt children to have to decide what makes sense. The word problems (1 - 4) above can all be adjusted to involve remainders in context.

References:

Bird, R. (2009) *Overcoming Difficulties with Number,* London:Sage

Haylock, D. (2006), *Mathematics Explained for Primary Teachers, (3rd edition),* London: Sage

Further reading:

Back, J. *Difficulties with Division* [online], Cambridge:NRICH
Available at: http://nrich.maths.org/5450 [Access Date: June 2013]

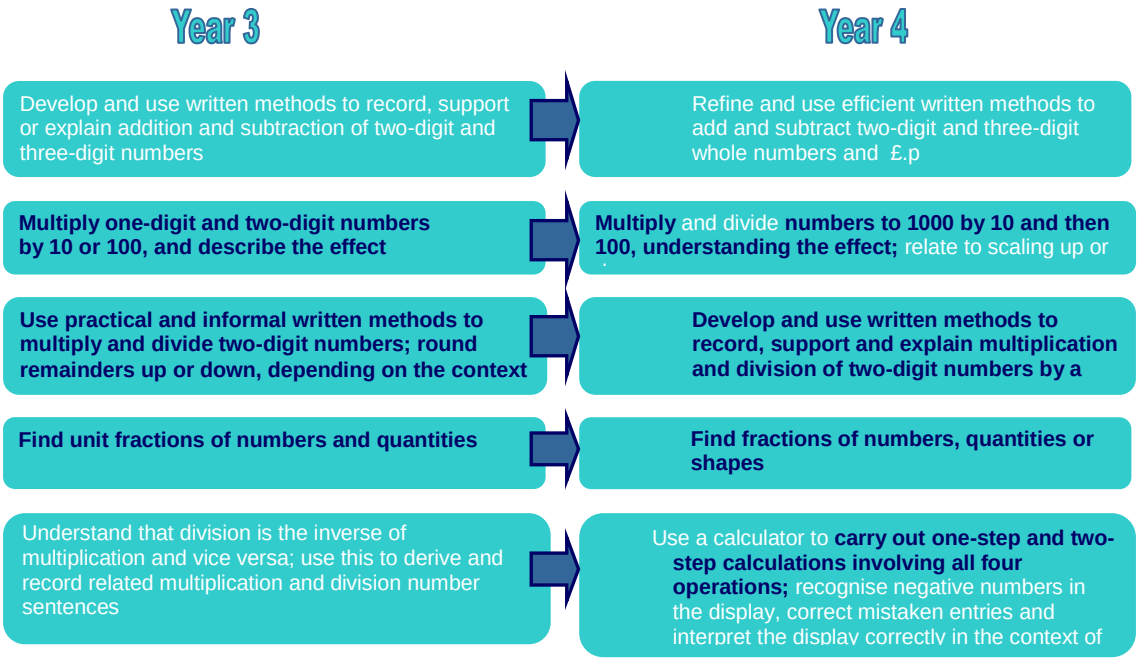
Key Language

Division, share, array, multiplication, inverse, divide, groups.

Resources

Numicon, number lines, Cuisenaire, multi link should be available for children to use if they choose, art straws are useful for fractions and decimals, multiplication squares, 100 squares

Learning objectives



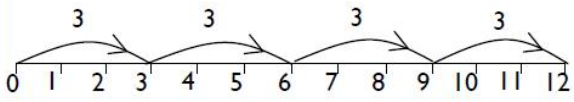
Calculation progression

Calculation Strategies

Division as grouping (repeated addition)

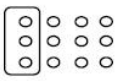
Keep adding 3 until you get to 12

12 ÷ 3 = 4

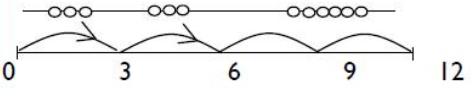


12 divided into groups of 3

or



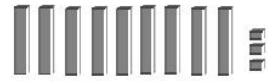
First group of 3



Use bead string in parallel with numberline

Division as sharing

93 ÷ 3 = 31



10 for you, 10 for you etc...

11 ÷ 5 = 2 r1

10 divided in to equal groups of 2

Grouping will lead to chunking later on

10 divided equally by 2

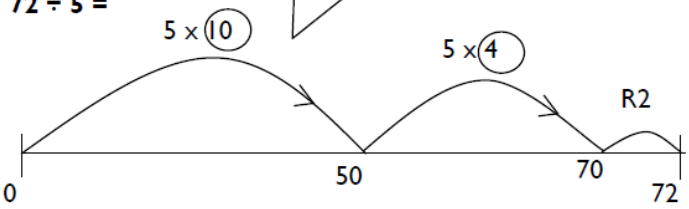
Calculation Strategies

Division as grouping

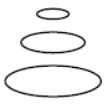
TU ÷ U

72 ÷ 5 =

Can we add ten lots of 5?



Chunking by adding chunks on a number line:
Supports multiplication.
Supports mental methods.



10 + 4 = 14 r2

Children can use this method to calculate some quotients mentally.

It is helpful to jot down some facts to help before you get started:

5 x 10 = 50
5 x 5 = 25
5 x 2 =10
5 x 3 = 15
5 x 4 = 20

Calculation progression 2

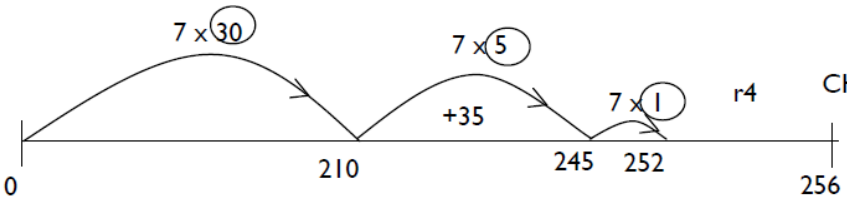
Calculation Strategies

Children adding more efficient chunks

1. Chunking on a number line

256 ÷ 7

Answer: 36 remainder 4
Or $36\frac{4}{7}$



CHECK (36 x 7) + 4 = 256

Progress to dividing by 2 digit numbers.

Children can use jottings to support the process, such as listing the first few multiples and x100, x50, x10 x20 etc for chunking.

2. Short division (bus stop) for ÷ U

256 ÷ 7

$$\begin{array}{r} 0\,3\,6 \\ 7\overline{)\,2\,{}^{2}5\,{}^{4}6} \end{array}$$ r4 or 4/7

It is important when first teaching this to ensure children understand the place value of the digits and link to chunking:

Commentary: Can I do any hundreds times 7? A: No, put a zero. Still have 256
Can I do any tens times 7? A: 30, with 4 tens left.
Can I do any ones times 7? A: 6, with 4 left.

Once the children are secure in their place value, they can start to treat the digits individually (how many 7s in 2? how many 7s in 25?...)

Calculation Strategies

'Bus stop' method for division of whole numbers and decimals by U

Starting with 3 digit whole numbers

567 ÷ 6

$$\begin{array}{r} 0\,9\,4\,.\,5 \\ 6\overline{)\,{}^{5}5\,6\,{}^{3}7\,.0} \end{array}$$

26.88 ÷ 7

$$\begin{array}{r} 0\,3\,.\,8\,4 \\ 7\overline{)\,2\,{}^{2}6\,{}^{5}.\,8\,{}^{2}8} \end{array}$$

3÷8

$$\begin{array}{r} 0\,.\,3\,7\,5 \\ 8\overline{)\,3\,{}^{3}.\,{}^{6}0\,{}^{4}0} \end{array}$$

Bus stop for division by TU

234 ÷ 13

$$\begin{array}{r} 0\,1\,8 \\ 13\overline{)\,2\,{}^{2}3\,{}^{10}4} \end{array}$$

13
26
39
52

Estimate first.
Check using the inverse.

Children are using jottings to support their calculations, such as the first few multiples of the divisor.

They can give the remainder in the form required (integer, fraction or decimal).

Assessment/self assessment

Try to answer every question. If you need more space, there is spare paper available. You can do whatever method you like to solve them.

(Teachers – q1-2 L3; q3-7 L4; q8-10 L5)

Q1. 28 children go swimming. They are put in groups of 4.

How many groups will there be?

Q2. Some children share 18 strawberries.

Each child gets 3 strawberries

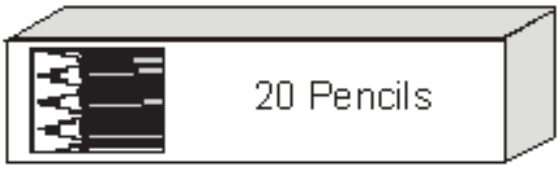
How many children are there?

children

Q3. 50 children need **two** pencils each.

There are 20 pencils in a box.

How many boxes of pencils are needed?



boxes

Q4. Calculate **56 ÷ 4**

Q5.

Alan has **45 beans**.

He plants **3 beans** in each of his pots.

How many pots does he need?

pots

Leila puts **4 seeds** in each of her pots. She uses **6 pots** and has **1 seed** left over.

How many seeds did she start with?

Q6. Sam **sold 10** of his old Playstation games in a car boot sale.

He made £30.

How much did he sell each game for?

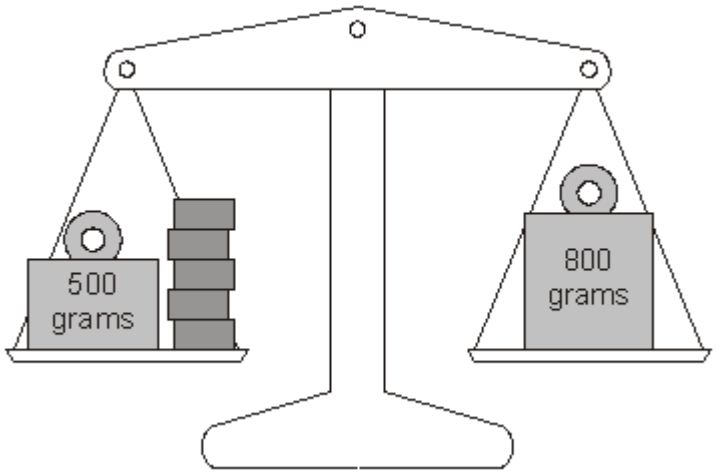
Q7. Circle the **two** divisions which have an answer of **5 remainder 2**

17 ÷ 5 **17 ÷ 3**

22 ÷ 4 **22 ÷ 5**

Q8. Lin has five blocks which are all the same.

She balances them on the scale with two weights.

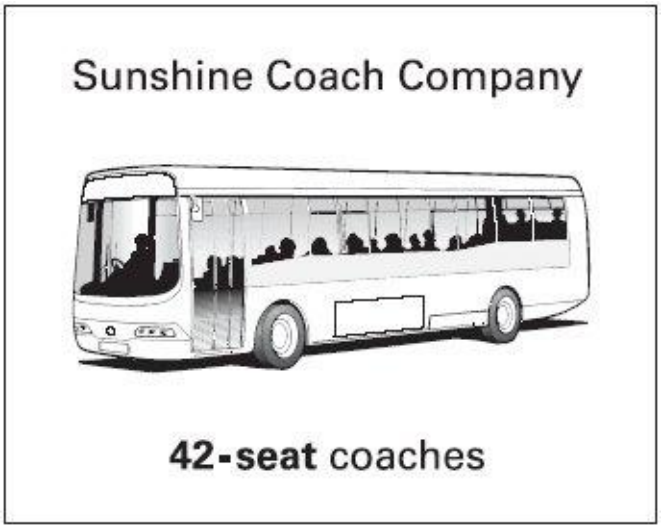


Calculate the weight of **one** block.

Show your **working**. You may get a mark

Q9. Calculate 942 ÷ 6

Q10.



272 children and **26** adults from Hill School go on a coach trip.

How many **42-seat** coaches does the school need to hire?

Planning

Planning to include:

- Short test base assessment task
- Planning to start with arrays and maths photos
- To move on to written method of division
- Investigations and problems solving
- With questions from N-Rich to ensure HA are suitably stretched
- Same assessment as start of unit.

Suggested activities

Please see attached Power Point with a wide range of activities

Key questions from N-Rich

Exemplifying, Specialising

Tell me about/ show/ choose/ Draw...
Is there another?
What's it like?
Give me an example of/ and another/ and another…
Is this a … ?
What makes a ...?
Can you find one that doesn't …?
Are there any special ones?

Generalising, Conjecturing

What is this an example of?
What happens in general?
Can you say why this is special?
What happened here? And here?
Can you see a pattern? Describe the pattern.
Is it always, sometimes, never …?
Describe all of these in a simple way that includes them all?

Completing, Deleting, Correcting

What can we change so that it stays true?
What can we change to make it false?
Tell me what's wrong with …
What needs to be changed so that…is true/false?

Comparing, Sorting, Organising

What's the same about …?
What's different about …?
Sort or organise these by …
Is it or is it not …?

Changing, Varying, Reversing, Altering

What happens if we change …?
What if …?
Here is an answer ..., what was the question?
Can you do it another way? And another? ...
Which is the quickest/easiest …?

Explaining, Justifying, Verifying, Convincing, Refuting

Explain why …
Give a reason (using or not using…)
How can we be sure that …?
Tell me what is wrong with …
Is it ever false that …? (always true that …?)
How is … used in …?
Explain the role/use of …
Convince yourself/ a friend/ your teacher

Adapted from Jeffcoat, M., Jones, M., Mansergh, J., Mason, J., Sewell, H. and Watson, A. (2004) *Primary Questions and Prompts*. Derby: Association of Teachers of Mathematics.